Solutions to JEE Main Home Practice Test - 10 | JEE - 2024

PHYSICS

SECTION-1

1.(B) The H like atom is in the third excited state i.e., $n = 4$.

Energy corresponding to this wave length $= \frac{12431 \times 51}{62000} = 10.2 eV$

This is the $E_2 - E_1$ for H and $E_4 - E_2$ for He^+ we get $Z = 2$ for $4 \to 2$ radiation.

Hence the atom is Helium ion.

2.(C) For minimum deviation

$$\sin\left(\frac{A+\delta_m}{2}\right) = \mu \sin\left(\frac{A}{2}\right)$$

$$\sin\left(\frac{60+\delta_m}{2}\right) = \sqrt{2}\sin 30° = \frac{1}{\sqrt{2}}$$

$$\left(\frac{60+\delta_m}{2}\right) = 45°$$

$$\delta_m = 30°$$

At minimum deviation, we known that angle of incidence and angle of emergency are equal $i = e$

$$\delta_m = 2i - A$$

$$i = \frac{\delta_m + A}{2} = \frac{30+60}{2} = 45°$$

3.(D) $\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$

Or $R^3 = nr^3$ or $R = n^{1/3} r$

$$R = 2n^{1/3} mm$$

Or $v_0 \propto r^2, v_0' \propto R^2$

Now, $\frac{v_0'}{v_o} = \frac{R^2}{r^2} = \frac{4n^{2/3}}{4}$

Now, $\frac{32}{8} = n^{2/3} \; or \; n^{2/3} = 4$

Or $n = 4^{3/2} \; or \; n^{2/3} = 4$

Or $n = 4^{3/2} = \sqrt{64}$

Or $n = 8$

4.(D) $\Delta Q = \Delta U + W$

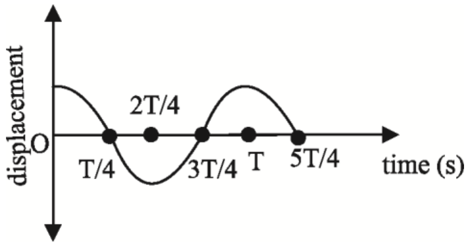
Since container is non conducting therefore

$$\Delta Q = 0 = \Delta U + W$$

$$W = -\Delta U = -n\frac{f}{2}R\Delta T = -2 \times \frac{3}{2}R(400-300)$$

$$= -3 \times \frac{25}{3} \times 100 J = -2500 J$$

5.(D) Given : A displacement time graph of a particle executing SHM.



The force on the particle at $t=\frac{3T}{4}$, acceleration at $t=T$, speed at $t=\frac{T}{4}$, t at which potential energy is same as kinetic energy.

From the graph, the equation of SHM is:

$$x=A\cos\omega t$$

I. At $t=\frac{3T}{4}$, the particle is at its mean position, so force on the particle is zero.

II. At $t=T$, the particle is at its extreme position, so acceleration is maximum.

III. At $t=\frac{T}{4}$, the particle is at its mean position, so speed of the particle is maximum.

IV. PE = KE

$$\frac{1}{2}kx^2=\frac{1}{2}k\left(A^2-x^2\right)$$

$$A^2=2x^2$$

$$x=\frac{A}{\sqrt{2}}=A\cos\omega t$$

$$\cos\omega t=\frac{1}{\sqrt{2}}$$

$$\omega t=\frac{\pi}{4}$$

$$\frac{2\pi}{T}t=\frac{\pi}{4}$$

$$t=\frac{T}{8}$$

So , statements A, B, C are correct.

6.(B)

7.(C) At $x=10\,m,\ U=84\,J$

At $x=-10\,m,\ U=164\,J$

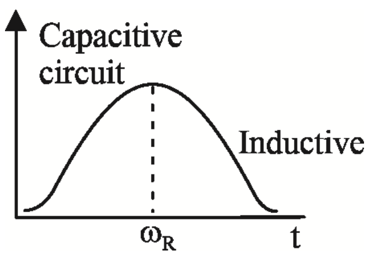
And at $x=2m\ \ U_{\min}=20J$

8.(D) $\omega_R=\frac{1}{\sqrt{LC}}$

$\omega_2<\omega_R$ from diagram

Circuit will behave as capacitance circuit

Current leads voltage.



9.(C) $F_e = mg\tan\theta$

$= \left(1.20\times10^{-3}kg\right)\left(10\,ms^{-2}\right)\tan 37^\circ = 0.0090\,N$

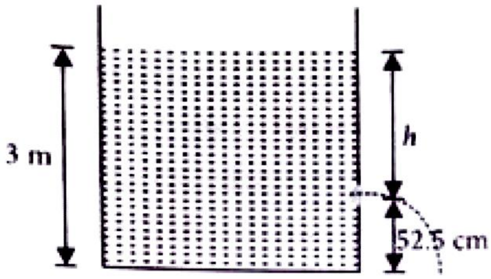
(Balance force in x- and y-directions.)

Also :

$$F_e = Eq = \frac{Vq}{d}$$

$$V = \frac{Fd}{q} = \frac{(0.009N)\times(0.0500m)}{9.0\times10^{-6}C} = 50.0V$$

10.(A) Suppose A be the area of cross section of tank, a be the area of hole, v_e be the velocity of efflux, h be the height of liquid above the hole. Let v be the speed with which the level decreases in the container. Using equation of continuity, we get



$$av_e = Av \text{ or } v = \frac{av_e}{A}$$

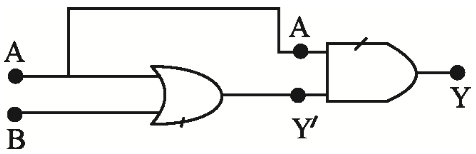
Using Bernoulli's theorem, we have

$$P_0 + h\rho g + \frac{1}{2}\rho v^2 = P_0 + \frac{1}{2}\rho v_e^2$$

$$h\rho g + \frac{1}{2}\rho\left(\frac{av_e}{A}\right)^2 = \frac{1}{2}\rho v_e^2$$

$$v_e^2 = \frac{2hg}{1-(a^2/A^2)} = \frac{2\times(3-0.525)\times10}{1-(0.1)^2} = 50m^2s^{-2}$$

11.(B)



Boolen expression

OR $\Rightarrow Y = A + B$

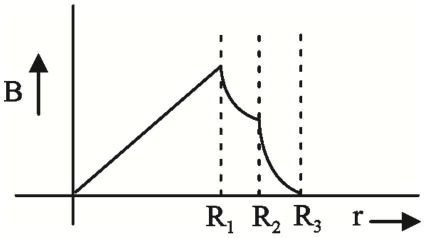
AND $\Rightarrow Y = AB$

$Y' = A + B$

Boolean expression for $Y = (Y'.A) = (A+B).A$

Option (A) is correct.

12.(C)

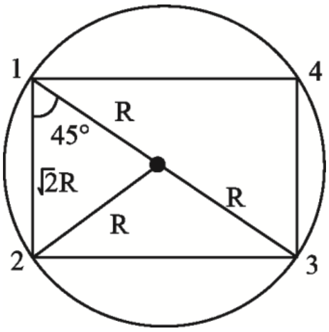


From Ampere's law, the field at the axis is zero. From $x = 0$ to R_1, the field increased linearly as the charge enclosed increases.

From $x = R_1$ to R_2 and from $x = R_2$ to R_3, the field decreases hyperbolically but with different slopes as the media are different.

Hence, the required graph is (c).

13.(D) $F_{12} = \dfrac{GM^2}{2R^2}$ $F_{14} = \dfrac{GM^2}{2R^2}$



The resultant of these two forces is $\left(\dfrac{\sqrt{2}GM^2}{2R^2}\right)$. Now, $F_{13} = \left(\dfrac{GM^2}{4R^2}\right)$

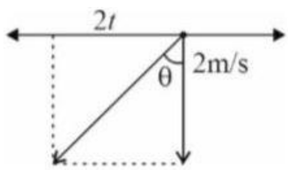
The combined resultant of all the forces is

$$\frac{\sqrt{2}GM^2}{2R^2} + \frac{GM^2}{4R^2} \; or \; \frac{GM^2}{R^2}\left[\frac{\sqrt{2}}{2} + \frac{1}{4}\right]$$

Equating this with centripetal force, we get $\dfrac{Mv^2}{R} = \dfrac{GM^2}{R^2}\left[\dfrac{2\sqrt{2}+1}{4}\right]$

Or $v^2 = \dfrac{GM}{R}\left[\dfrac{2\sqrt{2}+1}{4}\right] or \; v = \sqrt{\dfrac{GM}{R}\left(\dfrac{2\sqrt{2}+1}{4}\right)}$

14.(B) $v = u + at \; ; v_{boy} = 2t$

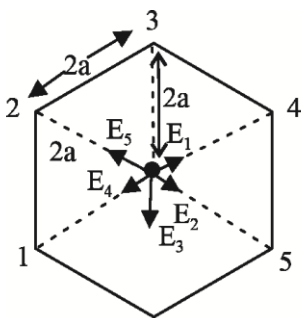


$$\tan\theta = \frac{2t}{2} \; ; \sec^2\theta \frac{d\theta}{dt} = 1$$

$$\frac{d\theta}{dt} = \cos^2\theta \; ; \frac{d\theta}{dt} = \frac{1}{1+t^2}$$

$$t = \frac{1}{2} \; ; \frac{d\theta}{dt} = \frac{1}{1+\left(\frac{1}{2}\right)^2} = \frac{4}{5} rad / s$$

15.(B)



$$E = \frac{kq}{(2a)^2}$$

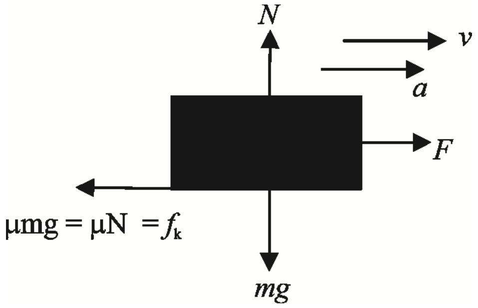
$$E_1 = E_2 = ... = E_5 = E$$

$$E_{net} = E_3 = \frac{kq}{4a^2} = \frac{q}{16\pi \in_0 a^2}$$

16.(D) $t = \dfrac{xy^2}{z^3}$

$$\frac{\Delta t}{t} = \left|\frac{\Delta x}{x}\right| + 2\left|\frac{\Delta y}{y}\right| + 3\left|\frac{\Delta z}{z}\right| = 1 + 2(3) + 3(2) = 1 + 6 + 6 = 13\%$$

17.(B) At any instant



Power delivered by this F is given as constant.

$P = FV \Rightarrow F = \dfrac{P}{V}$ (F is variable)

By NLM.

$F - f_k = ma$

$$\Rightarrow \frac{P}{V} - \mu mg = m\frac{dv}{dt} \qquad \Rightarrow \frac{P - \mu mg}{V} = m\frac{dv}{dt} \qquad \Rightarrow \frac{dv}{dt} = \left(\frac{P - \mu mgV}{mV}\right)$$

When $\dfrac{dv}{dt} = 0$ at that time $\dfrac{P - \mu mgV}{V} = 0 \Rightarrow V_{\max} = \dfrac{P}{\mu mg}$

18.(A) Energy of each satellite in the orbit $= \dfrac{-GMm}{2r}$

Total energy of the system before collision,

$$E_i = E_1 = E_2 = 2E = -2 \times \frac{GMm}{2r} = -\frac{GMm}{r}$$

As the satellites of equal mass are moving in the opposite directions and collide inelastically, the velocity of the wreckage just after the collision is

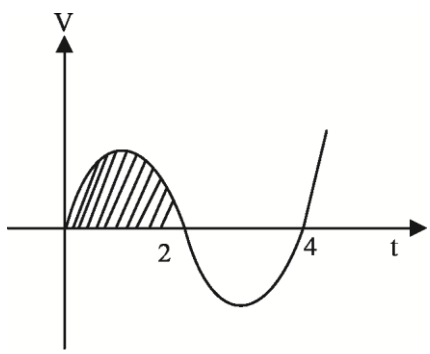
$mv - mv = 2mV, i.e., V = 0$

The energy of the wreckage just after the collision will be totally potential and will be

$$E_f = \frac{GM\,2m}{r} = -\frac{2GMm}{r}$$

As after collision the wreckage comes to standstill in the orbit, it will move along the radius towards the earth under gravity.

19.(C) $\dfrac{dv}{dt} = \dfrac{F_0}{m}\cos\dfrac{\pi t}{2} \Rightarrow \displaystyle\int_0^v dv = \frac{F_0}{m}\int_0^t \cos\frac{\pi t}{2}dt \Rightarrow v = \frac{2F_0}{\pi m}\sin\frac{\pi t}{2}$



Distance traveled from $t = 0$ to $t = 2\,\text{sec}$.

$$= \int_0^2 v\,dt$$

$$= \text{shaded area} = \frac{2F_0}{\pi m}\int_0^2 \sin\frac{\pi t}{2}dt = \frac{8F_0}{\pi^2 m}$$

20.(B) $R_{80°C} = R_{0°C}\left[1+\alpha\,\Delta T\right]$

$26.4\Omega = 20.0\Omega\left[1+\alpha\left(80-0\right)\right]$

$\frac{26.4}{20} = 1+80\alpha$

On solving, $\alpha = 4\times10^{-3}{}^{\circ}C^{-1}$.

SECTION - 2

1.(1) Kinetic energy = K

$-mgh = \Delta K$

$-mgh = \frac{K}{2} - K$

$mgh = \frac{K}{2}$

$2mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$4mgh = mv^2 + I\frac{v^2}{R^2}$ $\quad$ {Let $I = n\left(mr^2\right)$ }

$4mgh = mv^2 + nmR^2\frac{V^2}{R^2}$

$4mgh = mv^2 + nmv^2$

$40\times\frac{15}{4} = v^2\left(1+n\right)$

$15\times10 = 100\left(1+n\right)$

$\left(n+1\right) = \frac{15}{10} = \frac{3}{2}$

$n = \frac{1}{2}$

$\left(I = \frac{mR^2}{2}\right)$

$x = 1$

2.(4) No of waves $= \frac{thickness}{wavelength}$

Let x be the thickness of air column

So $\frac{x}{\lambda_{air}} - \frac{x}{\lambda_{vaccum}} = 2$

$\frac{x}{\lambda}\left[1.0003-1\right] = 2$

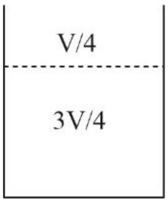
$x = \frac{2\times6000\times10^{-10}}{0.0003} = 4\,mm$

3.(171) On expansion moles of V/4 gas escapes

PV = nRT

Number of moles escaping (n')

$$P\left(\frac{V}{4}\right) = n'RT$$



$$\Rightarrow \; n' = \frac{n}{4}$$

At higher temperature (T') ($n - n'$ moles will occupy V volumes)

$$PV = (n - n')RT'$$

$$nRT = \left(n - \frac{n}{4}\right)RT' \Rightarrow T' = \frac{4}{3}T$$

$$T' = \frac{4}{3}(60 + 273) = 444\,K$$

= 171°C

4. (25) Effective resistance is 5Ω.

$$I = \frac{E}{R} = \frac{Blv}{R} \; or \; v = \frac{IR}{Bl} \quad \text{Or} \quad v = \frac{1\times10^{-3}\times5}{2\times10\times10^{-2}}\,ms^{-1} \quad \text{Or} \quad v = 0.025\,ms^{-1} = 25\,mms^{-1}$$

5.(160) Given $a = 8cm$ and $\omega = 2\pi n = 20\pi \; rad/s$

Let the phase constant be ϕ

The displacement equation can be written as $x = 8\sin(20\pi t + \phi)$

Given , at $t = 0; x = 4\,cm,$ therefore

$$4 = 8\sin(20\pi(0) + \phi) \Rightarrow \sin(\phi) = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{6}$$

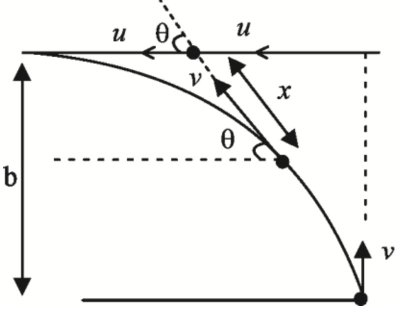
The displacement equation $x = 8\sin\left(20\pi t + \frac{\pi}{6}\right)$

Differentiating the above equation w.r.t time ‘t’

$$\frac{dx}{dt} = v = 160\pi\cos\left(20\pi t + \frac{\pi}{6}\right) \qquad \therefore\; v_{\max} = \pm160\pi\,cm/s\left[when\cos\left(20\pi t + \frac{\pi}{6}\right) = \pm1\right]$$

Value of x is 160

6.(1) Assume that water in river is at rest so Dock will move towards left with speed u and boat is continuously aiming at dock.



$$(v - u\cos\theta) = -\frac{dx}{dt}$$

$$-\int_b^0 dx = \int_0^T \left(vdt - u\cos\theta dt\right)$$

Initial separation is b.

$$b = vT - u\int_0^T \cos\theta dt \quad \text{........(i)}$$

$\because\ \theta$ is variable.

$$uT = \int_0^T v\cos\theta dt$$

Horizontal distance covered is same.

$$uT = v\int_0^T \cos\theta dt$$

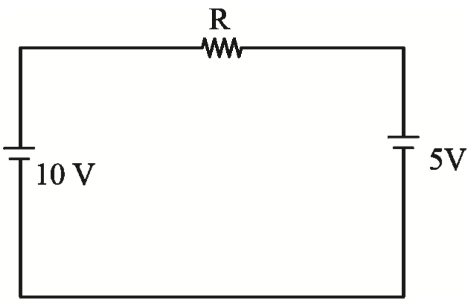
$$\frac{uT}{v} = \int_0^T \cos\theta dt$$ put in equation (i)

$$b = vT - u.\frac{uT}{v}$$

$$b = T\left(\frac{v^2 - u^2}{v}\right)$$

$$T = \frac{bv}{v^2 - u^2}$$

7.(5) For graph voltage at $t = 4\sec$ is 10 volt.



$$i = \frac{10-5}{1} = 5$$

8.(10) When the disc slides down and comes onto the plank, then

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} \quad \text{.........(i)}$$

Let v_1 be the common velocity of both the disc and plank when they move together. From law of conservation of linear momentum,

$$mv = (M + m)v_1$$

$$v_1 = \frac{mv}{(M+m)} \quad \text{...........(ii)}$$

Now, change in KE $=(K)_f-(K)_i=(work\ done)_{friction}$

$$\frac{1}{2}(M+m)v_1^2-\frac{1}{2}mv^2=(work\ done)_{friction}$$

$$W_{fr}=\frac{1}{2}(M+m)\left[\frac{mv}{M+m}\right]^2-\frac{1}{2}mv^2=\frac{1}{2}mv^2\left[\frac{m}{M+m}-1\right]$$

As , $\frac{1}{2}mv^2=mgh\ \therefore W_{fr}=-mgh\left[\frac{M}{M+m}\right]$

$$W_f=-5\times10\times3\left[\frac{10}{15}\right]=-100$$

Value of $x=10$

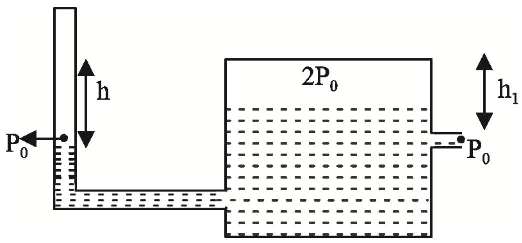
9.(42) Cylinder $A\rightarrow Q=nC_p\Delta T_1$ (isobaric process)

Cylinder $B\rightarrow Q=nC_v\Delta T_2$ (isochoric process)

$$C_p\Delta T_1=C_v\Delta T_2$$

$$\frac{7R}{2}\times30=\frac{5R}{2}\times\Delta T_2\ \Rightarrow\ \Delta T_2=42K$$

10.(1)



Pressure at same height is same, therefore $h=h_1$

$$x=1$$

CHEMISTRY

SECTION - 1

1.(B) $[Co(NH_3)_5Cl]\,Cl_2 \longrightarrow [Co(NH_3)_5Cl]^{2+} + 2Cl^-$

Moles of $[Co(NH_3)_5Cl]Cl_2 = \dfrac{0.1}{1000} \times 100 = 0.01$ mole

Moles of AgCl formed $= 2 \times$ moles of Cl^- in complex

$= 0.01 \times 2 = 0.02$ mol

2.(B)

3.(B) Exception

4.(B) Fajan's Rule

Polarisation $\propto$ Covalent character

5.(B) 

Electron withdrawing group

6.(C)

- Metallic radii increase in a group from top to bottom. Thus, Li < Na < K < Rb is true
- Electron gain enthalpy of Cl > F and decreases along a group. Thus, I < Br < F < Cl is true
- Ionisation enthalpy increases along a period from left to right but due to presence of stable half-filled orbitals in N, ionization enthalpy of N > O. Thus, B < C < N < O is incorrect

7.(B)

$NaBH_4$ reduces $-CHO$

H_2 / Pd reduces both $>C=C<$ and –CHO

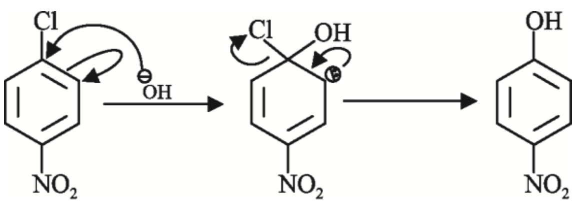
8.(B) $k = \dfrac{0.7}{2.1} h^{-1} = \dfrac{2.303}{7} \log \dfrac{10}{A_t}$

or $\log \dfrac{10}{A_t} = \dfrac{7 \times 0.7}{2.1 \times 2.303}$ or $\log \dfrac{10}{A_t} = 1.01 \quad (\simeq 1)$

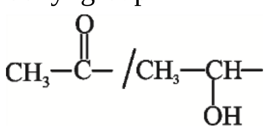
$\log \dfrac{10}{A_t} = 1$

$\dfrac{10}{A_t} = 10 \Rightarrow A_t = 1$

9.(C)



10.(A) 2, 4-D NP (+ve) $\Rightarrow$ Carbonyl group

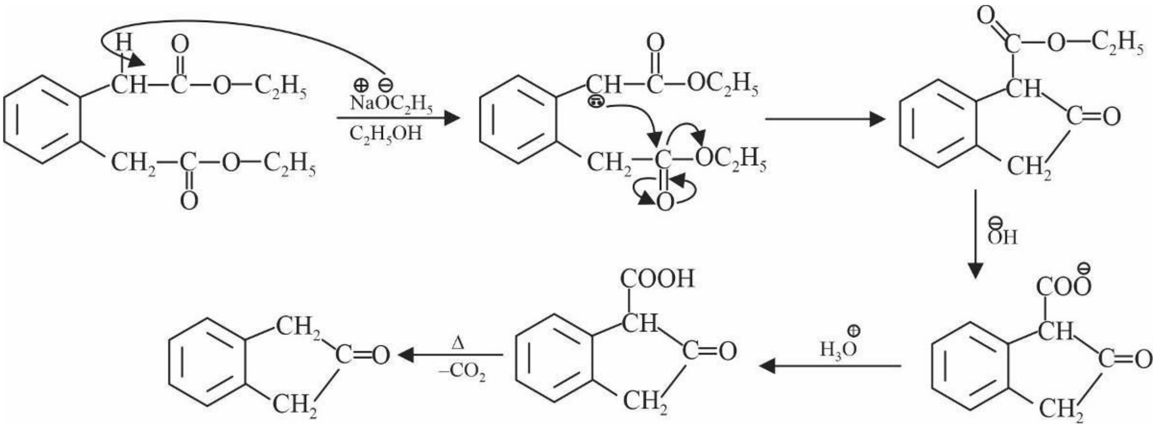
$I_2 / OH^- (+ve) \Rightarrow$ 

T.R (-ve) $\Rightarrow -CHO$ absent

11.(C) E_2 Mechanism

Anti elimination by $C_2H_5O^-$ $(-HBr)$

12.(B)



13.(B) With progressive increase in atomic number, the reduction potential of halogens decreases, thus oxidizing power also decreases.

Hence, a halogen with lower atomic number will oxidise the halide ion of higher atomic number and therefore, will liberate them from their salt solution. Hence, the reaction,

$Cl_2 + 2F^- \longrightarrow 2Cl^- + F_2$

is not possible.

14.(A) According to Kohlrausch's law

$\lambda_{eq}^{\infty} Na_2SO_4 = \lambda_{eq}^{\infty}(NaCl) + \lambda_{eq}^{\infty} K_2SO_4 - \lambda_{eq}^{\infty} KCl$

$= (123.7 + 152.1 - 147.0)\ \Omega^{-1} cm^2 eq^{-1}$

$= 128.8\ \Omega^{-1} cm^2 eq^{-1}$

15.(D) $P_4 + HNO_3 \rightarrow H_3PO_4 + NO_2 + H_2O$

16.(C) Monomer of cellulose

17.(C)

$\underset{\text{Yellow}}{CrO_4^{2-}} \underset{OH^-}{\overset{H^+}{\rightleftharpoons}} \underset{\text{Orange}}{Cr_2O_7^{2-}}$

18.(C) Leads to α-halo carboxylic acid, not acid halide.

19. (C) CH_3MgBr react with H_2O.

20.(B)

SECTION - 2

1.(495) $E = \dfrac{hc}{\lambda} = \dfrac{6.626 \times 10^{-34} \times 3 \times 10^8}{242 \times 10^{-9}}$ J / atom

$= \dfrac{6.626 \times 10^{-34} \times 3 \times 10^8 \times 6.023 \times 10^{23} \times 10^{-3}}{242 \times 10^{-9}}$ kJ / mole $= 494.73$ kJ / mole

2.(3) pH = 9.26 indicates $[NH_4OH] > [HCl]$

and thus mixture is a basic buffer since, HCl will react with equivalent amount of NH_4OH forming NH_4Cl.

Let HCl = x mL = x millimole

$NH_4OH = (300 - x)\,mL$

$= (300 - x)\,\text{millimole}$

NH_4Cl formed = x millimole

NH_4OH unreacted

$= 300 - x - x = (300 - 2x)\,\text{millimole}$

pOH = 14 – 9.26 = 4.74

$pK_b = 14 - 9.26 = 4.74$

$$pOH = pK_b + \log\frac{[NH_4^+]}{[NH_4OH]}$$

$$4.74 = 4.74 + \log\frac{x}{300 - 2x}$$

$$\frac{x}{300 - 2x} = 1$$

$x = 100\,mL = \text{volume of HCl}$

(300 – x) = 200 mL = volume of NH_4OH

Hence, volume ratio of NH_4OH and HCl

= 2 : 1

3.(2) I_3^- and XeF_2 has linear shape

4.(32) No. of half life in 50 minutes $= \frac{50}{10} = 5$

$$\frac{[A_t]}{[A_0]} = \frac{1}{2^n}$$

Where $n \rightarrow$ no. of half life

$$[A_t] = \frac{[A_0]}{2^5} = \frac{1}{32} \times [A_0]$$

Where $[A_0]$ -Initial concentration

$[A_t]$ -Concentration at any time ‘t’

5.(20) $(meq)_{H_3PO_4} = (meq)_{Ba(OH)_2}$

$1.5 \times V \times 3 = 0.5 \times 90 \times 2$

$$V = \frac{0.5 \times 180}{1.5 \times 3} = 20\,ml$$

6.(6) $\Lambda_{eq} = \frac{k \times 1000}{N}$

$$1.53 = \frac{3.06 \times 10^{-6} \times 1000}{N}$$

$N = 2 \times 10^{-3}$

Molarity $= \frac{\text{Normality}}{n - \text{factor}}$

For $BaSO_4$, n − factor = 2

$$M = \frac{2 \times 10^{-3}}{2} = 10^{-3}$$

$$\left[Ba^{+2}\right] = \left[SO_4^{2-}\right] = 10^{-3}$$

$$K_{sp} = \left[Ba^{+2}\right]\left[SO_4^{2-}\right] = 10^{-3} \times 10^{-3} = 10^{-6}$$

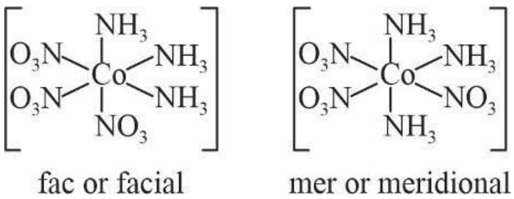
7.(4) $2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}$

Given, equilibrium amount of SO_2 and SO_3 is same

$$p_{SO_3} = p_{SO_2}$$

$$K_p = \frac{\left(p_{SO_3}\right)^2}{\left(p_{SO_2}\right)^2 p_{O_2}} \qquad p_{O_2} = \frac{1}{K_p} = \frac{1}{4}$$

8.(2) $\left[Co\left(NH_3\right)_3\left(NO_3\right)_3\right]$ has two geometrical isomers, fac or facial isomer and mer or meridional isomer.



9.(12) The expression for the depression in the freezing point and the molality is $K_f = \frac{\Delta T_f}{m}$

For a solution which freezes at $-0.22°C$, $(\text{molality})_f = \frac{x \times 1000}{100} = \frac{0.22}{K_f}$

For a solution which freezes at $-0.25°C$, $\left(\text{molality}\right)_f = \frac{x \times 1000}{w} = \frac{0.25}{K_f}$

On dividing equation (1) by equation (2), we get

$$\frac{0.22}{0.25} = \frac{w}{100}$$

Hence, $w = 88g$

The grams of ice that would have separated $100 - 88 = 12g$

10.(569) $\Delta G = \Delta H - T\Delta S$

For spontaneous process

$\Delta G < 0$

$$\therefore \quad T_{switch} = \frac{\Delta H}{\Delta S} = \frac{-33,000}{-58} K = 568.9K \simeq 569K$$

MATHEMATICS

SECTION-1

1.(C) $f(x) = \tan \lambda x$

$as\ f\left(\frac{\pi}{4}\right) = 0 \Rightarrow \tan\frac{\lambda\pi}{4} = 0$

$\lambda = 4n$

As K is an natural no.

$\Rightarrow f(k\pi) = \tan 4n\pi k = 0$

$$\sum_{k=1}^{10} \frac{1}{\cos k.\cos(k+1)} = \frac{1}{\sin 1}\sum_{k=1}^{10}\frac{\sin[(k+1)-k]}{\cos k\cos(k+1)}$$

$$= \frac{1}{\sin 1}\sum_{k=1}^{10}\tan(k+1) - \tan k = \frac{\tan 11 - \tan 1}{\sin 1} = \sin(10)\sec(11)\sec(1)\operatorname{cosec}(1)$$

2.(C)

x_i	f_i	$x_i f_i$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
3	p	3p	4	4p
4	2	8	1	2
5	3	15	0	0
7	q	7q	4	4q

$Mean = \frac{3p+8+15+7q}{p+q+5} = 5$

$2p - 2q = -2$

$p - q = -1$

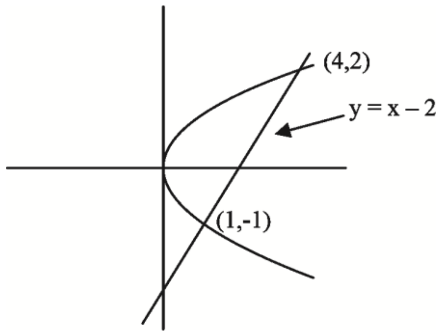
Variance $= (S.D)^2 = 2.2$

$\frac{4p+4q+2}{p+q+5} = 2.2 \Rightarrow p+q=5$

$p = 2, q = 3$

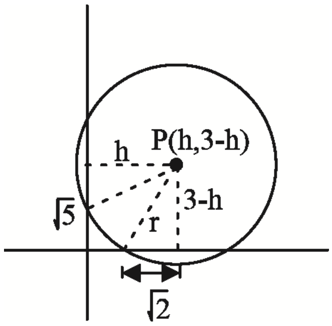
New mean $= \frac{6+8+15+24}{10} = \frac{53}{10} = 5.3$

3.(C)



$$Area = \int_{-1}^{2}\left[(y+2) - y^2\right]dy = \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_{-1}^{2} = \frac{9}{2}$$

4.(C)



$$r^2 = h^2 + 5 = (3-h)^2 + 2$$

$$\Rightarrow h = 1$$

$$r^2 = 1 + 5 \Rightarrow r = \sqrt{6}$$

5.(B) $2\vec{a} - \vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$

$\vec{a} - 2\vec{b} = -4\hat{i} + 4\hat{j} - 4\hat{k}$

Area of $= \frac{1}{2}\left|\left(2\vec{a} - \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right|$

$\left(2\vec{a} - \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right) = 3\hat{i} + 24\hat{j} + 15\hat{k}$

Area $= \frac{9\sqrt{10}}{2}$

6.(C) $2A = \tan\frac{\pi}{9} + 2\cot\frac{2\pi}{9}$

$2A = \cot\frac{\pi}{9}$ $\qquad \because \tan\theta + 2\cot 2\theta = \cot\theta$

$4A + \tan\frac{\pi}{18} = 2\cot\frac{\pi}{9} + \tan\frac{\pi}{18} = \cot\frac{\pi}{18}$

Also $\tan\frac{4\pi}{9} = \cot\frac{\pi}{18}$ $\quad \because \frac{4\pi}{9} + \frac{\pi}{18} = \frac{\pi}{2}$

$$\frac{\left(4A + \tan\frac{\pi}{18}\right)}{\tan\frac{4\pi}{9}} = 1$$

7.(B) Rationalizing numerator two time and denominator once

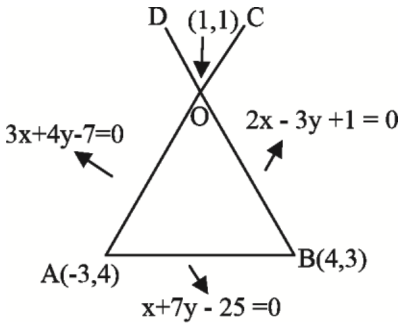
$$\lim_{x \to 0} \frac{x^2 \times 2\sin^6 x\left[\sqrt{1+\sin^8 x} + \sqrt{1-\sin^8 x}\right]}{2\sin^8 x\left[\sqrt[4]{1+\sin^6 x} + \sqrt[4]{1-\sin^6 x}\right]\left[\sqrt{1+\sin^6 x} + \sqrt{1-\sin^6 x}\right]} = \frac{1}{2}$$

8. (A) Mid point of AC i.e. centre of P of parallelogram $P(1,1)$

Equation of diagonal BD is equation of BP

i.e. $x - 2y + 1 = 0$ point $(3,2)$ lies on it.

9.(B)

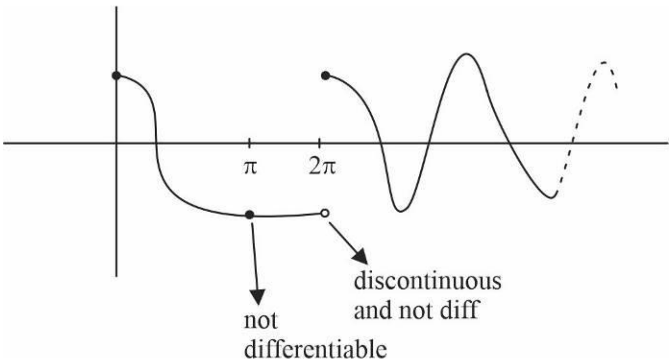


As D is the mid point of AC and BD

Point C is $(5,-2)$ and D is $(-2,-1)$

Equation of CD is $x+7y+9=0$

10.(B)



11.(A) $C:(x^2y^2-4x-8y-5)+\lambda(x^2+y^2-12x+2y+29)=0$

Centre : $\left(\dfrac{2+6\lambda}{1+\lambda},\dfrac{4-\lambda}{1+\lambda}\right)$

Common chord $4x-5y-17=0$

Centre lies on common chord

$\therefore\ \lambda=\dfrac{29}{12}$

$\therefore$ centre $\left(\dfrac{1+6(29/12)}{1+29/12},\dfrac{4-29/12}{1+29/12}\right)$

$\therefore$ centre $\left(\dfrac{198}{41},\dfrac{19}{41}\right)$

12.(B) $x\left(1-x^2\right)dy=y\left(1+x^2\right)dx$

$\dfrac{xdy-ydx}{x^2}=ydx+xdy$

$d\left(\dfrac{y}{x}\right)=d\left(xy\right)$

Integrating both sides

$\dfrac{y}{x}=xy+c$

$y=x^2y+cx$

$y = \frac{cx}{1-x^2}$

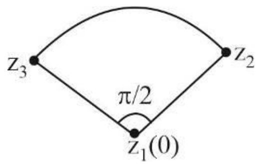
As $y(2) = 2 \Rightarrow c = -3$

$y(x) = \frac{-3x}{1-x^2}\, at\, x = 3,\ y(3) = \frac{9}{8}$

13.(B) Step (I) point P becomes, $P_1(-b,-a)$

(II) after translation $P_2(-b+3,-a)$

(III) Rotation of 90°



$1+i = [(-b+3)-ai][\cos 90° + i\sin 90°]$

$1+i = +a+(3-b)i$

$a = 1, b = 2$ value of $a+2b = 5$

14.(C) $a^{\log_a \sqrt[8]{9^{r-1}+2}} = \left(9^{r-1}+2\right)^{1/8} \quad \because\ a^{\log_a x} = x$

5^{th} term is ${}^{12}C_4\left[\left(9^{r-1}+2\right)^{1/8}\right]^8\left[\left(3^r+2\right)^{1/4}\right]^4 = 3135$

$\frac{12\times 11\times 10\times 9}{4\times 3\times 2\times 1}\times\frac{3^{2r}+18}{9}\times\left(3^r+2\right) = 11\times 285$

$3^r = 1$

$r = 0$

15.(D) $R = \{(x,y)\in Z\times Z : (x+y)(x-y) = 0\}$

Here $(x, y\in Z\times Z)$ if $x+y = 0\, or\, x-y = 0$

If $x = -y\, or\, x = y$

R is reflexive as $(x,x)\in Z\times Z$

Here $(x,y)\in Z\times Z$ if $x+y = 0\ or\ x-y = 0$

$\left.\begin{array}{l}\text{If } x = -y \text{ or } x = y\\ \text{If } y = -x \text{ or } y = x\end{array}\right] \Rightarrow x = |y| \text{ or } y = |x|$

$\Rightarrow (y,x)\in Z\times Z$

Let $(x,y)\in Z\times Z$ and $(y,z)\in Z\times Z$

$x = |y|$ and $y = |z|$

$x = |z|$

$(x,z)\in Z\times Z$

Hence R is an equivalence relation

16.(D) $f(x) = 3^{36\sin x - 27\cos x}.3^{36\cos^3 x - 48\sin^3 x}$

$= 3^{12\left[3\sin x - 4\sin^3 x\right]+9\left[4\cos^3 x - 3\cos x\right]}$

$f(x) = 3^{12\sin 3x + 9\cos 3x}$

$A = 3^{-15}, B = 3^{15}$

$-\frac{b}{a} = 3^{-1} + 3 = \frac{10}{3}$

$10a + 3b = 0$

17.(B) Let probability of winning toss in any match and losing a toss is W and L respectively then $W = L = \frac{1}{2}$

Here Probability of winning toss atmost 'n' times is

$= {}^8C_0 W^0 L^8 + {}^8C_1 W^1 L^7 + \ldots\ldots + {}^8C_N W^N L^{8-N} > \frac{1}{2}$

$\Rightarrow \frac{1}{2^8}\left({}^8C_0 + {}^8C_1 + \ldots\ldots + {}^8C_N \right) > \frac{1}{2}$

$N \geq 4$

Ans. 4

18.(C) **Let** $\frac{x-2}{1} = \frac{y-a}{3} = \frac{z-2}{2} = \lambda$

And $\frac{x+1}{2} = \frac{y-b}{3} = \frac{z-2}{1} = k$

$\lambda + 2 = 2k - 1, 3\lambda + a = 3k + b, 2\lambda + 2 = k + 2$

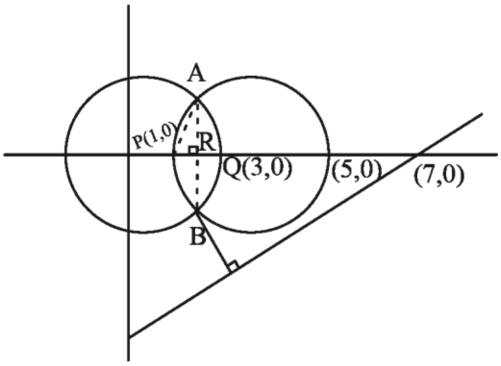
$\lambda = 1, k = 2$ and $a = b + 3$

Also $(\lambda + 2, 3\lambda + a, 2\lambda + 2) \equiv (3, 3 + a, 4)$ lies on $\frac{x-1}{2} = \frac{y-1}{4} = \frac{z}{4}$

$a = 2, b = -1$

so $2a - b = 5$.

19.(B)



z_1 represents points A and B.

In ΔARP

$AP^2 = PR^2 + AR^2$

$4 = 1 + AR^2$

$AR = \pm\sqrt{3}$

Print $A\left(2, \sqrt{3}\right)$, $B\left(2, -\sqrt{3}\right)$

Minimum distance will be distance of a from the line : $x - y = 7$

$AD = \left|\frac{2 + \sqrt{3} - 7}{\sqrt{2}}\right| = \frac{5 - \sqrt{3}}{\sqrt{2}}$.

20.(B) Here $g'(x) = f(x)$ (by N.L. theorem)

As $g(x) = 0$ has exactly 6 distinct roots in (m, n)

$f(x) = 0$ has atleast 5 distinct roots in (m, n)

$f'(x) = 0$ has atleast 4 distinct roots in (m, n)

$f''(x) = 0$ has atleast 3 distinct roots in (m, n)

$f(x).f'(x).f''(x) = 0$ has atleast 12 roots in (m, n)

SECTION-2

1.(2) dividing by x^2

$$x^2 - x - 4 - \frac{1}{x} + \frac{1}{x^2} = 0$$

$$x^2 + \frac{1}{x^2} - \left(x + \frac{1}{x}\right) - 4 = 0 \qquad \text{Put } x + \frac{1}{x} = t$$

$$t^2 - t - 6 = 0$$

$$t = 3, -2$$

$$x + \frac{1}{x} = 3, \; x + \frac{1}{x} = -2$$

$$x^2 - 3x + 1 = 0, \; x = -1$$

2 real positive roots.

2.(0) $\tan^{-1}\left(\frac{1}{2k^2}\right) = \tan^{-1}\left(\frac{k}{k+1}\right) - \tan^{-1}\left(\frac{k-1}{k}\right) \Rightarrow \lim_{n\to\infty} \sum_{k=1}^{n+1} \tan^{-1}\left(\frac{1}{2k^2}\right) = \lim_{n\to\infty} \tan^{-1}\left(\frac{n}{n+1}\right) = \frac{\pi}{4}$

3.(8) Apply $A.M. \ge G.M.$

$$\left(\frac{a}{b} + \frac{c}{d}\right) + \left(\frac{b}{c} + \frac{d}{a}\right) \ge 2\left\{\sqrt{\frac{ac}{bd}} + \sqrt{\frac{bd}{ac}}\right\}$$

$$\ge 2\left\{\frac{ac + bd}{\sqrt{abcd}}\right\}$$

$$\ge 2\left\{\frac{(a+c)(b+d)}{\sqrt{abcd}}\right\}$$

$$\ge 2\left\{\frac{2\sqrt{ac} \cdot 2\sqrt{bd}}{\sqrt{abcd}}\right\}$$

$$\ge 8$$

4.(7) $z = \frac{(4\sin\theta - i)}{(3 - i\sin\theta)} = \frac{(4\sin\theta - i)(3 + i\sin\theta)}{(3)^2 + \sin^2\theta}$

As given $z = \bar{z}$

$\operatorname{Im} g(z) = 0$

$\text{Im}\, g(z) = \dfrac{4\sin^2\theta - 3}{9+\sin^2\theta} = 0$

$\sin^2\theta = \dfrac{3}{4}$

$\theta = 60^\circ$, $\cos^2 3\theta = 1$

5.(40) To find divisor of form $2(2n+1)$

$N = 2^{10} \times 7^4 \times 5^3 \times 11^1$

No. of divisors $(1\times5\times4\times2) = 40$

6.(27) $(1+x)^n = 3 + \dfrac{8}{3} + \dfrac{80}{3^3} + \dfrac{240}{3^4} + ... = 1 + nx + \dfrac{n(n-1)x^2}{2} + ...$

On comparison, $n = -3$ and $x = \dfrac{-2}{3}$

7.(2240) $X = \{10, 11, 12, 13, ...150\}$

$Y = \{5, 9, 13, 17, 21, 25, ...\}$

$Z = \{5, 10, 15, 20, 25, ..., 149\}$

$(Y - Z) \cap X$ means those elements of Y which are present in Y but not in z and in the range of X.

Required sum $= (13+17+21+25+.....+145+149) - (25+45+65+.....+145)$

$= \dfrac{35}{2}\left[26+(34)\times4\right] - \dfrac{7}{2}\left[50+6(20)\right] = 2835 - 595 = 2240$

8. (5) $e^x(dy - dx) + e^{-x}(dy + dx) = 0$

$\left(e^x + e^{-x}\right)dy = \left(e^x - e^{-x}\right)dx$

$dy = \dfrac{\left(e^x - e^{-x}\right)}{\left(e^x + e^{-x}\right)}dx$

Integrating both sides

$y = \log_e\left(e^x + e^{-x}\right) + c$

At $x = 0, y = \log_e 2$

$c = 0$

$y = \log_e\left(e^x + e^{-x}\right)$

At $x = \ln 2$

$y = \log_e\left(2 + \dfrac{1}{2}\right) = \log_e \dfrac{5}{2}$

$e^{\log_e \frac{5}{2}} = \dfrac{5}{2}$

And $2e^{\log_e \frac{5}{2}} = 5$

9. (330) $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \quad \left(A^2\right)_{31} = 1+2$

$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix} \quad \left(A^3\right)_{31} = 1+2+3$

Similarly

$A^{10} = \begin{bmatrix} 1 & 0 & 0 \\ 10 & 1 & 0 \\ 55 & 10 & 1 \end{bmatrix} \quad \left(A^{10}\right)_{31} = 1+2+3+..........10 = \frac{10\times 11}{2} = 55$

$B = \begin{bmatrix} 10 & 0 & 0 \\ 1+2+3+...+10 & 10 & 0 \\ S & 1+2+3+.....+10 & 10 \end{bmatrix}$

where $S = 1+3+6+10+.....+55$

$S = 1+(1+2)+(1+2+3)+.....+(1+2+....+10)$

$t_n = 1+2+........+n = \frac{n}{2}(n+1)$

$S_n = \sum t_n = \frac{1}{2}\left(\sum n^2 + \sum n\right) = \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$

$B_{21} = 55, B_{32} = 55,\ B_{31} = 220$

$B_{21} + B_{31} + B_{32} = 330$

10.(3) $I = \int_0^{\pi/2} \left(\cos^3 x\right) e^{-\sin x} dx = \int_0^{\pi/2} (\cos x)\left(1-\sin^2 x\right) e^{-\sin x} dx$

Put $\sin x = t$

$\cos x dx = dt$

$\int_0^1 \underset{I}{\left(1-t^2\right)} \underset{II}{e^{-t}}\, dt$

$= \left[-\left(1-t^2\right)e^{-t}\right]_0^1 - 2\int \underset{I}{t}\ \underset{II}{e^{-t}}\, dt$

$= 1 - 2\int_0^1 t\, e^{-t}\, dt$

$m = 1,\ n = 2$

$m+n = 3$